1 J-integral and fracture toughness

A narrow strip of material with elastic modulus E, poisson's ratio ν and height h is rigidly attached to parallel platens, as shown in Figure 1. If the upper platen is displaced by δ, determine the geometric stress intensity factor for an edge crack between the platen faces.

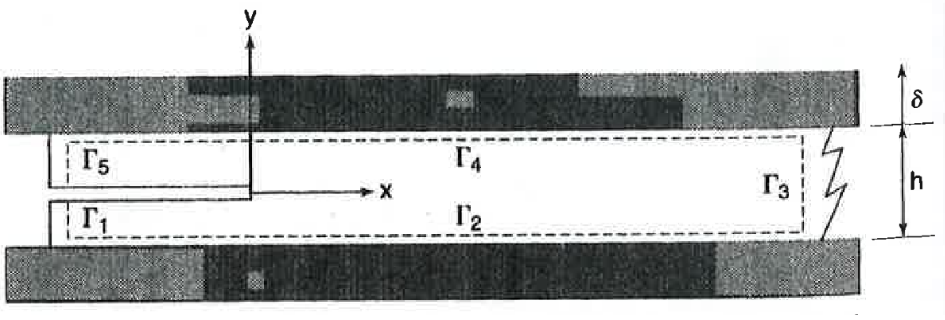

Figure 1: Edge crack in a strip of material.

Question 1

Decompose the J integral into subpart for each boundary of the contour Γ. Show that the value of J is only dependent on Γ_3. Show that:

$$J = \int_0^h W dy \tag{1}$$

Question 2

Using the geometry and Hooke's law, find the formula of ϵ_y and σ_y.
Hint: you might assume the problem is in plane stress conditions.

Question 3

Replace ϵ_y and σ_y in the J-integral and find the expression of J in terms of the displacement δ, the height h, the elastic modulus E and the poisson's ratio ν.

Question 4

Using the J-integral, find the expression of the stress intensity factor K.

Question 5

You want to design a joint with a material stress intensity factor K_C of 50 MPa.m$^{1/2}$. The material has a poisson's ratio of 0.3 and an elastic modulus of 200 GPa. Knowing that the maximum vertical elongation of the joint is 0.1 mm, what should be the height of the joint to resist crack propagation?

2 Numerical simulations of fracture

The following exercice has to be done using the Streamlit provided on the Moodle page. The goal of this exercice is to understand the capability of numerical simulations to simulate the fracture of materials. The numerical simulation will be done using the software Akantu, which is a Python-based software for finite element analysis.
The Streamlit is available under this link: `https://mse-424.streamlit.app/`

Question 1

Check the "plate-hole" example in the Streamlit. For this type of situation, what is the theoretical stress concentration factor (SCF) at the hole?

Question 2

Using the Streamlit, simulate the plate-hole example. Take the initial geometry ($a = b = 1$, $w = 10$, $l = 5$, $F = 10$). $h1$ is the mesh size around the hole, $h2$ is the largest mesh size. Do you observe the same SCF as the theoretical one?

Question 3

Try to reduce the size of the hole by a factor of 10, and enlarge the plate geometry by a factor of 2. Does the SCF change? Why?

Question 4

Check the "stress-intensity-factor" example in the Streamlit. Knowing that the Young's modulus is $E = 1$ GPa, what is the formula to get the far-field stress σ ?

Question 5

Using the the initial length of the crack $l = 10$, calculate the K_I stress intensity factor (take into consideration a semi-infinite plate with a crack). Now, check the values of K_I Akantu find by doing some curve-fitting into the numerical results. Are they close? If not, why?
Note that r_{fit} is the interval of the curve-fitting used to calculate the K_I stress intensity factor from the numerical results.

Question 6

Try to increase the size of the plate to $L = 30$, $W = 60$, and change the mesh size to $h1 = 0.02$. How is the K_I stress intensity factor now? Is it close to the theoretical value?

Question 7

Why is the K curve not following exactly the one obtained by Akantu ?
Hint: Check the log-log plot.

Question 8

What would happen to the log-log plot of the stress if we had a plastic zone, assuming the plastic zone is small enough so we can still consider a K-controlled regime?